

# On the cosmological evolution of $\alpha$ and $\mu$ and the dynamics of dark energy

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We study the cosmological evolution of the fine structure constant,  $\alpha$ , and the proton-to-electron mass ratio,  $\mu = m_p/m_e$ , in the context of a generic class of models where the gauge kinetic function is a linear function of a quintessence-type real scalar field,  $\phi$ , described by a Lagrangian with a standard kinetic term and a scalar field potential,  $V(\phi)$ . We further assume that the scalar field potential is a monotonic function of  $\phi$  and that the scalar field is always rolling down the potential. We show that, for this class of models, low-redshift constraints on the evolution of  $\alpha$  and  $\mu$  can provide very stringent limits on the corresponding variations at high-redshift. We also demonstrate that these limits may be relaxed by considering more general models for the dynamics of  $\alpha$  and  $\mu$ . However, in this case, the ability to reconstruct the evolution of the dark energy equation of state using varying couplings could be seriously compromised.

## I. INTRODUCTION

Variations of  $\alpha$  have been constrained over a broad redshift range ( $z = 0 - 10^{10}$ ) using various cosmological observations and laboratory experiments. The earliest constraints come from primordial nucleosynthesis which requires the value of  $\alpha$  at  $z \sim 10^{10}$  to be within a few percent of its present day value [1, 2, 3] (although tighter constraints can be obtained for specific models [4, 5, 6, 7, 8]). The cosmic microwave background temperature and polarization anisotropies give a constraint of comparable magnitude at much smaller redshifts  $z \sim 10^3$  [2, 9, 10, 11, 12].

At lower redshifts the situation is still controversial. A number of results, obtained through the measurement of the relative shifts of quasar spectral lines, suggest a cosmological variation of  $\alpha$  and  $\mu$  in the redshift range  $z = 1 - 4$  at about the  $10^{-5}$  level [13, 14, 15, 16]. However other analysis have found no evidence for such variations [17, 18, 19]. This situation should be resolved in the next few years in particular with the next generation of high resolution spectrographs such as ESPRESSO planned for ESO's Very Large Telescope (VLT) which will be a stepping stone towards the CODEX spectrograph planned for the European Extremely large Telescope (E-ELT) [20].

At even lower redshifts laboratory experiments at  $z = 0$  provide strong limits on  $\alpha$  variability  $|\dot{\alpha}/\alpha| = -2.6 \pm 3.9 \times 10^{-16} \text{ yr}^{-1}$  [21] while the constraints coming from the Oklo natural nuclear reactor limit the variation of  $\alpha$  in the redshift range  $z = 0 - 0.2$  to be less than one part in  $10^7$  [22] assuming that only  $\alpha$  has varied over time. Future laboratory tests will greatly improve current constraints. For example, the ACES (Atomic Clock Ensemble in Space) project will be able to constrain  $\dot{\alpha}/\alpha$  at the  $10^{-17} \text{ yr}^{-1}$  level [23]. However, even more spectacular bounds (up to  $10^{-23} \text{ yr}^{-1}$  [24]) may be available

in the not too distant future.

On a more theoretical front, it was realized that in models where the quintessence field is non-minimally coupled to the electromagnetic field [25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37] the dynamics of  $\alpha$  is directly related to the evolution of the scalar field responsible for the dark energy. It was shown [38], that for a broad class of models, varying couplings may be used to probe the nature of dark energy over a larger redshift range than that spanned by standard methods (such as supernovae [39, 40, 41, 42] or weak lensing [43]). Furthermore, it was claimed [38] that a high-accuracy reconstruction of the equation of state may be possible all the way up to redshift  $z \sim 4$ .

Throughout this paper we shall neglect the spatial variations of  $\alpha$  and  $\mu$  which is usually a good approximation [37, 44]. These may be relevant in the context of chameleon-type models [45, 46] where masses and coupling constants are strongly dependent on the local mass density or if there are domain walls separating regions with different values of the couplings [47]. However, in general, the late-time variation of the fundamental couplings is negligible in these models and consequently we shall not consider them further in this paper.

This paper is organized as follows. In Section II we shall consider a broad class of models for the evolution of  $\alpha$  and  $\mu$  where the gauge kinetic function is a linear function of a quintessence-type real scalar field described by a Lagrangian with a standard kinetic term and a scalar field potential,  $V(\phi)$ . We also assume that the scalar field potential is a monotonic function of  $\phi$  and that the scalar field is always rolling down the potential. We show how low redshift observations can lead to very stringent constraints on the dynamics of  $\alpha$  and  $\mu$  at high redshifts, for models within this class. In section III we consider an even more general class of models where we relax one or more of the above assumptions and discuss the possible impact of this generalization on our ability to reconstruct the dark energy equation of state using varying couplings. Finally we conclude in Section

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IV with a brief summary of our results and a discussion of future prospects. Throughout this paper we shall use fundamental units with  $\hbar = c = G = 1$  and a metric signature  $(+, -, -, -)$ .

## II. DYNAMICS OF $\alpha$ AND $\mu$

In this section we shall consider a class of models described by the action

$$S = \int d^4x \sqrt{-g} \mathcal{L}, \quad (1)$$

where  $\mathcal{L}$  is the Lagrangian for a real scalar field  $\phi$  coupled to the electromagnetic field with

$$\mathcal{L} = \mathcal{L}_\phi + \mathcal{L}_{\phi F} + \mathcal{L}_{\text{other}}, \quad (2)$$

where

$$\mathcal{L}_\phi = X - V(\phi), \quad (3)$$

$$X = \frac{1}{2} \nabla^\mu \phi \nabla_\mu \phi, \quad (4)$$

$$\mathcal{L}_{\phi F} = -\frac{1}{4} B_F(\phi) F_{\mu\nu} F^{\mu\nu}, \quad (5)$$

$B_F(\phi)$  is the gauge kinetic function,  $F_{\mu\nu}$  are the components of the electromagnetic field tensor and  $\mathcal{L}_{\text{other}}$  is the Lagrangian density of the other fields. The fine-structure constant is then given by

$$\alpha(\phi) = \frac{\alpha_0}{B_F(\phi)} \quad (6)$$

and, at the present day, one has  $B_F(\phi_0) = 1$ .

The equation of motion for the field  $\phi$  is

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{dV}{d\phi} - \frac{\alpha_0}{4\alpha^2} \frac{d\alpha}{d\phi} F_{\mu\nu} F^{\mu\nu}, \quad (7)$$

where a dot represents a derivative with respect to physical time,  $H = \dot{a}/a$  and  $a$  is the scale factor. The time variations of the fine structure constant induced by the last term on the r.h.s. of Eqn. (7) are very small (given Equivalence Principle constraints [26]) and can be neglected. Hence, throughout this paper we shall assume that the dynamics of  $\phi$  is fully driven by the scalar field potential,  $V(\phi)$  (and damped by the expansion).

We will, for the moment, assume that the gauge kinetic function is a linear function of  $\phi$  so that one has

$$\frac{\Delta\alpha}{\alpha} = \beta\Delta\phi, \quad (8)$$

where  $\Delta\alpha = \alpha_0 - \alpha$ ,  $\Delta\phi = \phi_0 - \phi$ ,  $\beta$  is a constant and we have also taken into account that  $\Delta\alpha/\alpha \ll 1$  (at least for  $z < 10^{10}$ ). We also assume that the scalar field potential,

$V(\phi)$ , is a monotonic function of  $\phi$  and that the field  $\phi$  is always rolling down the potential. If this is the case, and given a fixed value of  $\phi_0$ , then  $|\Delta\phi(z)| = |\phi_0 - \phi(z)|$  is maximized for a flat potential (here  $z = 1/a - 1$  is the redshift). Note that if  $dV/d\phi = 0$  then the dynamics of the scalar field  $\phi$  is simply given by

$$\ddot{\phi} + 3H\dot{\phi} = 0 \quad (9)$$

and, consequently,  $\dot{\phi} = \dot{\phi}_0 a^{-3}$ . For a non-flat monotonic potential  $|\dot{\phi}|$  cannot increase so rapidly with redshift and so

$$\dot{\phi} = \dot{\phi}_0 a^{-3s(a)} \quad (10)$$

with  $s \leq 3$ . Note that, in this case, the contribution of the damping term due to the expansion of the universe is attenuated by the driving term due to the potential  $V(\phi)$ . Hence, given a fixed value of the kinetic energy of the scalar field  $\phi$  at the present time its kinetic energy at  $z > 0$  will always be smaller than the corresponding value in the flat potential case. We may now calculate the value of  $\Delta\phi(z) = \phi_0 - \phi(z)$  for this special model (characterized by  $dV/d\phi = 0$ ) thus constraining the maximum allowed variations of  $\alpha$  as a function of  $z$ . For  $z < z_{\text{eq}}$  one has  $a \sim (t/t_0)^{2/3}$  and

$$\begin{aligned} f(z) &\equiv \frac{\Delta\alpha(z)}{\dot{\alpha}_0 t_0} = \frac{\Delta\phi(z)}{\dot{\phi}_0 t_0} = \frac{1}{\dot{\phi}_0 t_0} \int_t^{t_0} \dot{\phi} dt' \\ &= \frac{3}{2} \int_a^1 u^{-5/2} du = (1+z)^{3/2} - 1. \end{aligned} \quad (11)$$

If  $z > z_{\text{eq}}$  then  $a \sim (t_{\text{eq}}/t_0)^{2/3} (t/t_{\text{eq}})^{1/2}$  and

$$\begin{aligned} f(z) &= 2(1+z_{\text{eq}})^{-1/2} \int_a^{a_{\text{eq}}} u^{-2} du + \frac{3}{2} \int_{a_{\text{eq}}}^1 u^{-5/2} du \\ &= 2(1+z_{\text{eq}})^{-1/2} (z - z_{\text{eq}}) + (1+z_{\text{eq}})^{3/2} - 1 \end{aligned} \quad (12)$$

Here, we have assumed a sharp transition from the radiation to the matter-dominated era and we have neglected the small period of dark energy domination around the present time. This has a negligible impact on our results and greatly simplifies the calculations.

We use the values  $z_{\text{eq}} = 3200$  and  $t_0 = 13.7$  Gyr consistent with latest WMAP 5-year results [48]. Note that at  $f(z = 4) \sim 10$ ,  $f(z = 10^3) \sim 3 \times 10^4$  and  $f(z = 10^{10}) \sim 4 \times 10^8$ . We thus see that a constraint on the value of  $\dot{\alpha}_0 t_0 / \alpha_0$  at  $z \sim 0$  of about 1 part in  $10^7$  consistent with no variation is enough to either rule out all current positive results for the variation of  $\alpha$  or the broad class of varying  $\alpha$  models presented above. On the other hand, low redshift constraints at the level of 1 part in  $10^7$  or less will beat present CMB results at constraining the value of  $\alpha$  around the recombination epoch (note that this level of precision will be within reach of the ACES project). Although even better constraints are needed in order to put useful bounds on the value of  $\alpha$  at the nucleosynthesis epoch we should bear in mind that spectacular

improvements may be expected in not too distant future [24, 49]. For example, Flambaum [24] has claimed that an improvement in the precision up to  $10^{-23} \text{ yr}^{-1}$  (equivalent to a constraint of about 1 part in  $10^{13}$  in  $\dot{\alpha}_0 t_0 / \alpha_0$ ) may be possible using the effect of the variation of  $\alpha$  on the very narrow ultraviolet transition between the ground state and the first excited state in  $^{299}\text{Th}$  nucleus. If, in the future, we are able to achieve this level of precision and find a negative result for the variation of  $\alpha$  then the values of  $\alpha$  at recombination and nucleosynthesis would respectively have to be within about  $10^{-8}$  and  $10^{-4}$  of the present day value, a level of precision that cannot be easily achieved by other means.

The relation between the variations of  $\alpha$  and  $\mu$  is model dependent but, in general, we expect that

$$\frac{\dot{\mu}}{\mu} = R \frac{\dot{\alpha}}{\alpha}, \quad (13)$$

where  $R$  is a constant. The value of  $R$  is of course model dependent (see [4, 50, 51, 52, 53, 54, 55] for a more detailed discussion of specific models) but if  $|R|$  is large then variations of  $\mu$  may well be easier to detect than variations of  $\alpha$ , a fact pointed out and studied in detail in ref. [38].

### III. MORE GENERAL MODELS

In the previous section we considered a class of models with  $\mathcal{L}(X, \phi) = X - V(\phi)$ . In this section we consider an even more generic class of models with a real scalar field  $\phi$  governed by an arbitrary Lagrangian of the form  $\mathcal{L}(X, \phi)$ . Its energy-momentum tensor may be written in a perfect fluid form

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu - pg^{\mu\nu}, \quad (14)$$

by means of the following identifications

$$u_\mu = \frac{\nabla_\mu \phi}{\sqrt{2X}}, \quad \rho = 2Xp_{,X} - p, \quad p = \mathcal{L}(X, \phi). \quad (15)$$

In Eq. (14),  $u^\mu$  is the 4-velocity field describing the motion of the fluid (for timelike  $\nabla_\mu \phi$ ), while  $\rho$  and  $p$  are its proper energy density and pressure, respectively. The equation of motion for the scalar field is now

$$\tilde{g}^{\mu\nu} \nabla_\mu \nabla_\nu \phi = \frac{\partial \mathcal{L}}{\partial \phi}, \quad (16)$$

where

$$\tilde{g}^{\mu\nu} = p_{,X} g^{\mu\nu} + p_{,XX} \nabla^\mu \phi \nabla^\nu \phi. \quad (17)$$

An example of an algebraically simple but physically interesting class of Lagrangians is  $\mathcal{L}(X) = f(X) - V(\phi)$  with  $f(X) \propto X^n$ . If the scalar field potential vanishes ( $V = 0$ ) then  $w = 1/(2n - 1)$  and consequently when  $n = 1$  we have a standard massless scalar field,

$n = 2$  corresponds to background radiation and in the limit  $n \rightarrow \infty$  the scalar field describes pressureless non-relativistic matter. If the scalar field,  $\phi$ , is homogeneous then  $X = \dot{\phi}^2/2$  and its dynamics is given by

$$n2^{1-n}(\dot{\phi})^{2n-2} \left( (2n-1)\ddot{\phi} + 3H\dot{\phi} \right) = -\frac{dV}{d\phi}, \quad (18)$$

so that

$$\dot{\phi} = \phi_0 a^{-3/(2n-1)}, \quad (19)$$

if  $dV/d\phi = 0$ . This is hardly surprising since for a constant  $w$  the evolution of the energy density with the scale factor is given by  $\rho = wX^n \propto a^{-3(w+1)} = a^{-6n/(2n-1)}$ . If  $n > 1$  then the scalar field evolves more slowly with redshift than in the  $n = 1$  case and consequently the constraints considered in the previous section still apply here. However, if  $1/2 < n < 1$  then the scalar field may evolve much more rapidly than with  $n = 1$  and the above constraints may no longer be valid. We shall not consider models with  $n < 1/2$  since, in this case, the sound speed squared,  $c_s^2 = p_{,X}/\rho_{,X}$ , is negative and consequently the solutions are unstable with respect to high frequency perturbations.

We will now, for the sake of illustration, consider the evolution of  $w$  for a family of models characterized by  $\mathcal{L}(X) = f(X) - V(\phi)$  with  $f(X) \propto X^n$  in two limiting cases: case I -  $|\ddot{\phi}| \ll 3H|\dot{\phi}|/(2n-1)$  and case II -  $|\ddot{\phi}| \gg 3H|\dot{\phi}|/(2n-1)$ . Let us start with case I for which it is a good approximation to set  $\ddot{\phi} = 0$  in Eq. (18), so that  $\dot{\phi} = \text{constant}$ . In this case

$$\rho + p = 2n(\dot{\phi}^2/2)^n = \rho_0 + p_0 = (1 + w_0)\rho_0, \quad (20)$$

and consequently

$$w(a) = \frac{p}{\rho} = \frac{p_0 + \Delta V}{\rho_0 - \Delta V} = \frac{w_0 + \Delta V/\rho_0}{1 - \Delta V/\rho_0}, \quad (21)$$

with  $\Delta V = V_0 - V$ . Also, we can show, using Eq. (18) and the condition  $\ddot{\phi} = 0$ , that  $\Delta V = -C\Delta \ln a = C \ln a$  where  $C = 3n2^{1-n}\dot{\phi}^{2n} = 3(1 + w_0)\rho_0$  and we have taken  $a_0 = 1$ . The evolution of the equation of state is then given by

$$w(a) = \frac{w_0 + 3(1 + w_0) \ln a}{1 - 3(1 + w_0) \ln a}. \quad (22)$$

Hence, we find no  $n$  dependence in this limit.

In case II with  $|\ddot{\phi}| \gg 3H|\dot{\phi}|/(2n-1)$  the energy density  $\rho$  is approximately conserved and, to a good approximation, the equation of state parameter is simply given by

$$w(z) = -1 + \frac{2n}{\rho} \left( \frac{\dot{\phi}^2(z)}{2} \right)^n = -1 + (1 + w_0) \left( \frac{\dot{\phi}(z)}{\dot{\phi}_0} \right)^{2n}. \quad (23)$$

Consequently, in this limit, the evolution of  $w$  with redshift, for a given evolution of  $\phi$ , is strongly dependent on

$n$ . Hence, if future constraints rule out the class of models described in the Section II then the ability to reconstruct the equation of state of dark energy from varying couplings would be compromised since to a given evolution of  $\phi$  there may be many different possible evolutions for the equation of state (given fixed values for  $\rho_0$  and  $w_0$ ).

Of course, there are other possible generalizations to the class of models introduced in the previous section. For example, we could relax the assumption that the gauge kinetic function is a linear function of  $\phi$ . Then the dynamics of  $\alpha$  would no longer need to be identical to that of  $\phi$  and could even be very different from it. However, if we allow for an arbitrary gauge kinetic function we may no longer be able to use cosmological limits on the evolution of  $\Delta\alpha/\alpha$  (or  $\Delta\mu/\mu$ ) with redshift,  $z$ , to constrain the evolution of  $\phi$ . Consequently, the ability to reconstruct the equation of state of the dark energy would again be seriously compromised.

On the other hand, if the gauge kinetic function is linear in  $\phi$  and  $\mathcal{L}(X, \phi) = X - V(\phi)$  we can, in principle, reconstruct the dark energy equation of state without further assumptions about  $V(\phi)$ . It may even be possible that future observations of the evolution of  $\alpha$  (or  $\mu$ ) with redshift require that  $\dot{\alpha}$  (or  $\dot{\mu}$ ) changes sign and lead us to consider non-monotonic potentials (see for example [56]). However, such models will have to be fine-tuned in order to give an equation of state parameter  $w \sim -1$  near the present time. Furthermore, it would be virtually impossible to determine whether the observed evolution of the couplings was due to special features of the scalar field potential or to a more complex kinetic term or gauge kinetic function.

## IV. CONCLUSIONS

In this paper, in section II, we considered a generic class of models for the evolution of  $\alpha$  and  $\mu$ . We then introduced a criterium that can be used to relate the limits on  $\Delta\alpha/\alpha$  (or  $\Delta\mu/\mu$ ) at different redshifts, for models within this class.

We have demonstrated that low-redshift constraints on the evolution of  $\alpha$  and  $\mu$  can provide stringent limits on the corresponding variations at high-redshift. In particular, a constraint on the value of  $\dot{\alpha}_0 t_0/\alpha_0$  at the  $10^{-7}$  level (within reach of the ACES project) would, if consistent with no variation, be able to rule out all current positive results for the variation of  $\alpha$ .

We have also shown that future constraints at  $z = 0$  may lead to limits on  $\Delta\alpha/\alpha$  at  $z = 1000$  which can be up to five orders of magnitude stronger than the best limits expected from future CMB experiments (such as Planck). At the nucleosynthesis epoch,  $z = 10^{10}$ , the limits will be weaker by about 4 orders of magnitude. Still, if an improvement up to  $10^{-23} \text{ yr}^{-1}$  in the measurement precision of  $\dot{\alpha}_0 t_0/\alpha_0$  is obtained in the future then zero redshift constraints could lead to more stringent limits on the variation of  $\alpha$  from  $z \sim 10^{10}$  than the ones imposed by the observed light element abundances.

On the other hand, we have shown that if future observations lead us to adopt more general models, such as the ones studied in section III, then the above constraints can be relaxed. However, in this case we may no longer be able to trace the dynamics of dark energy using varying couplings.

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